# UNIVERSITY OF KERALA
## Model Question Paper
## First Degree Programme in Geology
## Semester IV    Complementary Course for Geology
## MM 1431.3     Mathematics – IV
## (Vector Analysis, Fourier Series and Fourier Transforms)

**Time: 3 hours** **Maximum Marks: 80**

**Section-I**

**All the first 10 questions are compulsory. They carry 1 mark each.**

1. Give the natural domain of the vector valued function $\vec{r}(t) = t\,\hat{\imath} + \frac{1}{t}\hat{\jmath} + 3/t^3\hat{k}$
2. Find the arc length parametrization of the curve $x = t, y = t, z = t$ that has the same direction as the given curve and has the reference point $(0, 0, 0)$.
3. Suppose that during a certain time interval a proton has a displacement of $\Delta\vec{r} = 0.7\,\hat{\imath} + 2.8\,\hat{\jmath} - 1.5\,\hat{k}$ and its final position vector is known to be $\vec{r} = 3.6\,\hat{k}$. What was the initial position vector of the proton?
4. Determine the constant $a$ so that the vector $\vec{F} = (x + 2y)\,\hat{\imath} + (3y + 2z)\hat{\jmath} + (2y - az)\hat{k}$ is solenoidal.
5. If $V$ is the volume enclosed by a surface $S$, then find the value of $\iint_S \vec{r}.\vec{n}\,dS$
6. Using Stoke's theorem find the value of $\int_C \vec{r}\,.\,d\vec{r}$ where $C$ is a simple closed curve in 2-space.
7. Write down the Euler formulae for calculating the Fourier coefficients.
8. What is the standard form of Fourier series for an even function?
9. What are the sufficient conditions for the existence of Fourier transform.
10. State the convolution theorem.

**Section-II**

**Answer any 8 questions from among the questions 11 to 22.**
**These questions carry 2 marks each.**

11. A bug, starting at the reference point (1, 0, 0) of the curve $\vec{r} = \cos t\ \hat{\imath} + \sin t\,\hat{\jmath} + t\,\hat{k}$ walks up the curve for a distance of 10 units. What are the bug's final co-ordinates?
12. Define the inverse square law and show that such a field is solenoidal.
13. If $\vec{r} = x\,\hat{\imath} + y\hat{\jmath}$, show that $\nabla f(\vec{r}) = \frac{\vec{r}}{r}$ where $r = \sqrt{x^2 + y^2}$
14. Find the value of $n$ for which $r^n\vec{r}$ is irrotational.

15. Evaluate $\nabla \times (\vec{a} \times \vec{r})$ where $\vec{a}$ is a constant vector and $\vec{r}$ is the position vector of an arbitrary point in 3-space.
16. Using Green's theorem evaluate $\int_C f(x)dx + g(y)dy$ where $C$ is an arbitrary simple closed curve in an open connected set $D$. What do you infer about the vector field $\vec{F}(x,y) = f(x)\,\hat{\imath} + g(y)\,\hat{\jmath}$
17. Evaluate the flux of the vector field $\vec{F}(x,y,z) = z\,\hat{k}$ across the outward oriented sphere $x^2 + y^2 + z^2 = a^2$.
18. Find the nonzero function $f(x)$ such that $\vec{F}(x,y) = xy\ \hat{\imath} - x^2\,\hat{\jmath}$ is conservative.
19. Find the work done by the force field $\vec{F}(x,y) = f(x)y\ \hat{\imath} - 2xf(x)\,\hat{\jmath}$ on a particle that moves along the parabola $x = y^2$ from (0,0) to (1,1).
20. Find the Fourier series of $f(x) = x,\ 0 < x < 2\pi$
21. Find the Fourier series of $f(x) = |x|, -2 < x < 2$.
22. Derive the Fourier transform of $f'(x)$, the derivative of $f(x)$

**Section-III**

**Answer any 6 questions from among the questions 23 to 31.**

**These questions carry 4 marks each.**

23. Using the vector equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, find the curvature of the ellipse at the end points of the major and minor axes. Deduce the curvature of the circle.
24. Suppose that a particle moves through 3-space so that its position vector at time $t$ is $\vec{r} = t^2\,\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$. Find the scalar tangential and normal components of acceleration at time $t$. Also find the vector tangential and normal components of acceleration at time t = 1.
25. Prove that $curl(grad\ \varphi) = 0$ where $\varphi$ is a scalar point function.
26. Evaluate the surface integral $\iint_\sigma xz\,dS$ where $\sigma$ is the part of the plane $x + y + z = 1$ that lies in the first octant. What happens if the integrand is $xy$?
27. Evaluate using Green's theorem, $\oint_C y^2dx + x^2dy$ where $C$ is the square with vertices: $(0,0), (1,0), (1,1)$ and $(0,1)$ oriented counter clock-wise.
28. Verify Green's theorem for $f(x,y) = y^2 - 7y,\ g(x,y) = 2xy + 2x$ and $C$ is the circle $x^2 + y^2 = 1$.
29. Check whether $\vec{F}(x,y) = ye^{xy}\,\hat{\imath} + xe^{xy}\hat{\jmath}$ is conservative or not. If it is so, find the corresponding scalar potential.
30. Find the half range cosine series of $f(x) = \begin{cases} \frac{2K}{L}x; & 0 < x < L/2 \\ \frac{2K}{L}(L-x); & L/2 < x < L \end{cases}$

31. Find the Fourier transform of $e^{-x^2/2}$. What is your inference?

**Section-IV**

**Answer any 2 questions from among the questions 32 to 35.**

**These questions carry 15 marks each.**

32. A shell is fired from ground level with a muzzle speed of $320\ ft/s$ and elevation angle $60°$. Find the parametric equations for the shell's trajectory, the maximum height reached, the horizontal distance travelled and the speed of the shell at impact.

33. Consider the function $\vec{F}\,(x, y, z) = (x^2 - yz)\,\hat{\imath} + (y^2 - zx)\,\hat{\jmath} + (z^2 - xy)\hat{k}$ over the volume enclosed by the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b$ and $0 \le z \le c$. Verify Gauss's divergence theorem for $\vec{F}$.

34. Find the Fourier series of $f(x) = \frac{x^2}{2}; -\pi < x < \pi$ . Hence deduce: $\sum_{n=1}^{\infty} \frac{1}{n^2}$

35. Find the Fourier transform of $f(x)$, where

$$f(x) = \begin{cases} e^x; & x < 0 \\ e^{-x}; & x > 0 \end{cases}$$

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