FIFTH SEMESTER B.TECH DEGREE EXAMINATION

(2013 SCHEME)

13.506.3(ELECTIVE l) MATHEMATICAL METHODS IN CHEMICAL ENGINEERING (H)

(Model Question Paper)

Time: 3Hours Max.Marks:100

PART A

Answer **all** questions. **Each** question carries **2** marks.

1. Distinguish between finite and infinite dimensional spaces.
2. State the axioms to be satisfied by metric of a vector.
3. Are the following sets of vectors form a basis for R^3. Justify?

   $[1\ \ 0\ \ 1]^t$, $[2\ \ 1\ \ -1]^t$, $[3\ \ 2\ \ 1]^t$, $[5\ \ 9\ \ 10]^t$
4. Classify the following equations as parabolic, elliptic or hyperbolic.

   (a) $\frac{\partial u}{\partial t} = \frac{\partial 2u}{\partial x 2} + \frac{\partial 2u}{\partial y 2}$

   (b) $\frac{\partial u}{\partial x} + \frac{\partial 2u}{\partial x \partial y} + \frac{\partial 2u}{\partial x 2} + \frac{\partial 2u}{\partial z 2} + \frac{\partial u}{\partial y} = 0$
5. Explain Sturm- Louiville theory.
6. Discuss about the physical significance of green's function.
7. Determine the direction of fluid flow (laminar) between two horizontal plates when the pressure decreases with z. the governing equation is given by.

$$\frac{\partial 2\mathrm{vz}}{\partial 2\mathrm{y}2} = \frac{\rho}{\mu}\frac{\partial p}{\partial z}.$$

8. Classify the steady states of a two dimensional system in terms of its eigenvalues.
9. Discuss briefly on poincare maps.
10. Write short note on bifurcation theory. **(10x2 = 20 Marks)**

PART B

Answer **any one** question from **each** module.

Module – I

11. a) Consider two vectors, $A=(1\ 2\ 3)^T$ and $B=(1\ 0\ -1)^{T.}$

Find (i) metric between A and B; (ii) Norm of A and norm of B; (iii) Inner product of A and B. **10**

b) Check whether the following vectors are linearly dependent or independent.

$u_1 = [\ 1\ 1\ 1\ ]^t$, $u_2 = [\ 2\ 3\ 4\ ]t$; $u_3 = [\ 4\ 5\ 6\ ]^t$. **10**

OR

12. Check $u_1= [1\ 2\ 3]^t$, $u_2= [2\ 3\ 1]^t$, $u_3= [3\ 1\ 2]^t$ form a basis set or not. Using these basis vectors obtain a set of orthonormal basis set. Consider the fourth vector $u_4= [4\ 1\ -2]^t$ and express it in terms of the orthonormal basis vectors. **20**

Module – II

13. The dynamics of the system is given by $\frac{dx}{dt} = Ax + b$

$A= \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix}$ $b=[\ 1\ \ \alpha\ ]^t$

The system is at steady state with $\alpha = 0$. Calculate the evolution of x with time and the new steady state when α is changed to 3. **20**

OR

14. Consider

$$Lu = \frac{d2u}{dx2} - \frac{du}{dx},\ u(0) = 0,\ u(1) = 0.$$

(a) Find L* and B*

(b) Convert it to sturm- louiville form. **20**

Module III

15. Solve the non-homogeneous problem using green's function.

$\frac{d2u}{dx2}$ = sinx, Subject to u(0) =1, u(1) =2. **20**

OR

16. u''(x) = sin (πx) sin (2πx)
Subject to
u(0) = 0, u'(0) = u'(1).
Does this equation have a unique solution? Use: (a) Maximum principles (b) energy methods. **20**

Module IV

17. Determine the possible steady states and examine their stability for
$\frac{dx1}{dt} = x_1^2 - ax_1x_2 - x_1$ and $\frac{dx2}{dt} = bx_2^2 - x_1x_2 - 2x_2$ where a and b are real positive. Evaluate conditions on parameters such that saddle node and Hopf bifurcation occur in each steady state. **20**

OR

18. a) Give an account of the stability tests for n – dimensional equations. **10**
b) Discuss the methods to quantify the long-time system behaviour. **10**